RAMAKRISHNA MISSION VIDYAMANDIRA
(Residential Autonomous College under University of Calcutta)
B.A./B.SC. FIFTH SEMESTER EXAMINATION, DECEMBER 2012
THIRD YEAR
PHYSICS (Honours)
Paper : V
Date : 17/12/2012
Time : 11 am – 1 pm
Full Marks : 50

[Use separate Answer Book for each group]

Group - A

(*Answer* ***any three*** *questions*)

1. a) A block of mass m slides on the smooth surface of a wedge of mass M which slides on a smooth horizontal surface. Use Lagrange's equations to deduce— (i) the acceleration of the wedge, and (ii) the acceleration of the block relative to the wedge (Wedge angle is α) [5]

   b) Consider the two-particle system of masses M and m interacting by gravitational force. Using suitable generalized coordinates q_α, write down the Lagrangian L of the system. Hence calculate the corresponding generalized momenta p_α. Verify that the quantity $E = \sum p_\alpha \dot{q}_\alpha - L$ is a constant of motion. [5]

2. a) i) Define the Poisson Bracket (PB) of dynamical variables U=U(q,p), V=V(q,p)

   ii) If U = U(q,p) and H the Hamiltonian, show that $\frac{dU}{dt} = [U, H]_{PB}$

   iii) Hence show that if U and V are constants of motion then their PB, $[U,V]_{PB}$ is also a constant of motion. [6]

   b) Evaluate the Poisson brackets : (i) $[L_x, y]$ (ii) $[L_x, L_y]$, where $\vec{L} = \vec{r} \times \vec{p}$ is the angular momentum [4]

3. A rigid body has an axial symmetry (i.e. is a symmetric top) about a principal axis. Set up the Euler's equations for the force free motion of this body. Obtain the general solution of these equations, and give a geometrical interpretation of these results. [10]

4. a) Assume that the potential energy of a bi-atomic molecule is given by

   $U(r) = \frac{a}{r^5} - \frac{b}{r^3} (a > 0 \; ; \; b > 0)$ [where r is the distance between the atoms]

   i) Find the equilibrium point and the force constant between the two atoms.

   ii) What is the oscillation frequency of the molecule if the mass of each atom equals m? [2+3]

   b) Two identical simple harmonic oscillators along x-axis are coupled by a spring of spring constant K. For small oscillations along the x-axis, obtain the normal frequencies and the normal coordinates. [5]

5. a) Prove that the generalized momentum conjugate to a cyclic coordinate is conserved. [2]

   b) A particle is moving near the surface in the earth's gravitational field. Write down the Hamiltonian and hence, the equation of motion of the particle. (Neglect Earth's rotation) Are there any cyclic coordinates? Explain. [2+1+2]

   c) Find the Hamiltonian corresponding to the Lagrangian $L = a\dot{x}^2 + b\dot{y}^2 - Kxy$ (a, b, K are constants) [3]

Group – B

(*Answer* ***any two*** *questions*)

6. a) Derive the relativistic law of addition of velocities. Hence show that the law is in conformity with one of the basic postulates of special theory of relativity. [4+1]

   b) A particle has a velocity 6×10^7 m/sec in the X-Y plane at an angle of 60° with X-axis in the system S. Determine the magnitude and direction of its velocity in S´, when S´ has a velocity 3×10^7 m/sec along the positive X-axis. [3]

   c) What is the importance of negative result of Michelson-Morley experiment? [2]

7. a) If T be the relativistic kinetic energy of a particle and p the magnitude of its 3-momentum, find the rest mass energy of the particle in terms of p and T. [5]

   b) Any quantity which is left unchanged by the Lorentz Transformation is called a Lorentz scalar or invariant. Show that ΔS is a Lorentz scalar where $(\Delta S)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta z)^2$. Here Δt is the interval between two events and $[(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2]^{\frac{1}{2}}$ is the distance between them in the same inertial system. [5]

8. a) Show that D'Alembertian operator $\Box^2 = \nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}$ is invariant under Lorentz transformation. [4]

   b) Show that in Minkowski space, the four-velocity satisfies the relation $u_\mu u_\mu = -c^2$ (where the symbols have their usual meanings) [3]

   c) A $\pi^\pm$ mesons of mass m_π comes to rest and disintegrates into a $\mu^\pm$ meson of mass m_μ and a neutrino of zero rest mass. Find the kinetic energy of motion of the $\mu^\pm$ mesons. [3]